8 God and Infinity

8.1 Mathematics and Infinity

God is often associated with the Infinite. I argue that God is not infinite, although God may be known by thought processes using the Infinite. By means of the creation we can know God as object among other objects. This means that God is conceivable and correctly known by means of the conceivable. I have argued that God is the only object that can be conceived as pure existent. That is, as Creator, God exits before the creation as pure, not-described existent. Before the creation there is no way to discern between objects because there is only one object, God. All that can be said about God is that God is one and that God exists. I claim that God exists because we know that there is something rather than nothing, therefore, at least one thing exists. Before knowledge is possible through the creation, God is the only indescribable existent.

With the creation, discernment between objects and the resulting knowledge becomes possible. Hence, God the pure existent becomes describable; we have a language about God.

With respect to Anselm's Definition, infinity is an important concept because the thought sequence generated in the search to define God can involve an infinite, non-terminating sequence.

Mathematics provides a language for handing the idea of the infinite. Using the natural numbers $N = \{1,2,3,\dots\}$ where each number has a successor, we can say that whatever number is presented we can always produce its successor.

Therefore, the sequence never ends and we have a potentially infinite sequence. Peano’s axioms give us a basis for a more logical understanding of the natural number system. The axioms are as follows

1. There is a number zero ($0 \in N$).
2. For each number n there is a successor n’=(n+1) to n ($n' \in N$).
3. Zero is not a successor number ($n' \neq 0$).
4. If m, n are numbers then if m’= n’ then m = n.
5. If $A \subset N$ where $0 \in A$ and $n' \in A$ for every $n \in A$ then $A \subseteq N$.

Peano’s axioms are a way to describe the natural numbers but they are not a proof of the infinite nature of the natural. A proof using infinity is Euclid’s proof of the infinity of the prime numbers, which is as follows. Suppose that there is not an infinity of primes. Let the finite list of k primes be $p_1, p_2, p_3, p_4, \ldots, p_k$. Form a product of the finite list of k primes $p_1p_2p_3p_4 \ldots p_k$. Form the number $P = p_1p_2p_3p_4 \ldots p_k + 1$. Now P is not divisible by any of $p_1, p_2, p_3, p_4, \ldots, p_k$. Because any number > 1 is divisible by some prime, and P is not divisible by any of the listed primes, then P must be divisible only by itself, which means that it is another prime. When P is added to the list of primes the same procedure may be repeated producing another number divisible only by itself and not divisible by any of the previous primes. Therefore the number of primes is not complete.

There are two ways by which we can approach the infinite. One way is to speak of potential infinity and the other way is to speak of actual infinity. This recognises the two positions by which one can regard an infinite sequence. One position is from within the sequence, the other position is from outside the sequence. When one is within the sequence we have potential infinity because the sequence is never completed. This is because, by definition, there is always a successor term. Being within the sequence is the constructivist position, whereby it is claimed that you can only have as much of a sequence as you can construct, which implies that infinity is only ever approached as potentially the case. Cantor used the idea of a completed infinity, whereby one is outside the sequence. In this case the sequence may be viewed as an interval or a set of points. This also implies that the interval itself can be regarded as an object in a sequence. Hence we have the development of transfinite arithmetic which is the arithmetic based on the sequence of completed infinite quantities. The possibility of such a sequence is denied by the constructivist and the Intuitionist.

Dedekind defined a set as infinite if it was equivalent (equi-numerous) to a subset of itself. This allows for intervals of the number line to be mapped into sub-intervals of the number line (or the continuum). Cantor developed the theory of ordinal numbers and cardinal numbers. These were obtained from any collection of objects. If the nature of the objects were removed but not their order then the ordinals were obtained and if the order of the items was removed the cardinals were obtained. Two sets have the same cardinal number if there is a bijection between them. Cantor called the first infinite cardinal $\aleph_0$ (aleph-null). Given objects in a collection that retain their order, an ordinal number is seen as a union of all previous ordinal numbers. The ordinal numbers are ordered by inclusion ($\subseteq$). Cantor called the first infinite ordinal ω.

I want to make some comments about thought and infinity. This is in relation to the creation. Let me first say something about creation and infinity. I claim that the creation, which consists of discernible, existing objects, is finite. In the potential infinite sense, the creation cannot be infinite because it is never completed. Creation, when seen as a construction, is a sequence of created objects. If this were an infinite sequence it would never be completed, by definition. God does not go on creating things forever. An infinite process of creation implies that God cannot complete the creation.

Alternatively in the actual infinity sense, creation can be viewed as completed. There are two number models relevant here: the model of the countable numbers and the model of the non-countable numbers. A countable set of objects can be placed in a 1-1 correspondence with the natural numbers. Such a set of objects is not dense but it may be infinite. The objects of creation are not dense, because there is not always another object between any two objects. Discernment between objects is essential, because of the identity of indiscernibles. If we add that even God cannot complete an infinite construction (because by definition it is incompletable), then we are left with the creation as a finite collection of discernible objects.

However, the possibility of dealing with the creation as a completed infinity arises if we think of the conceivable creation. The creation is a collection of objects with relations between them. Language deals with the relations between objects. Language allows for the possibility of relations between objects that may not be the case in reality (the creation). I am not arguing that there is a separate conceivable world of which the actual world is an instance, because there is nothing prior to the creation and the creation consists of the objects created and the relations between them. Language (and thought) is possible because of the relations between objects, the ability to isolate (conceptually) these relations and substitute different objects as the terms of the relations. A horse is a discernible object with relations within the concept of the horse and with objects outside the horse. A unicorn is also perfectly and consistently conceivable. I have no idea why there are horses and not unicorns. There are narwhals with tusks and horses without tusks. Maybe there is a survival argument as to why some animals with tusks survive and some animal with tusks do not survive, but such arguments simply are stating that what is the case is the case and, in my opinion, have no explanatory power. Is the reason that there are no unicorns because the unicorns that did exist did not survive? My argument is that there are no unicorns because God did not create any. How does the 'survival of the fittest' argument explain that something does not exist? If existence is a function of fitness or its lack, does that imply that what does not exist, once did exist but did not survive? What did exist in order that what now exists may exist? The 'survival of the fittest' argument is simply stating that the fittest will survive and that those that survive will be the fittest. Which comes first, the capacity for fitness or the capacity for survival? If survival is the result of fitness then what is fitness the result of? Survival? Creation says that what actually is, is the result of divine fiat, functioning within the relations (rules) of the creation. This allows for adaptation within species but not across species, which would supposedly generate different species.

However, conceivable objects may be dense. Conceivably, between any two conceivable objects may be another conceivable object. So the model of the continuum or the real number interval containing a completed infinite sequence is relevant.

I wish to relate ideas about infinity to the realm of thought. I am seeking to show ways in which thought can lead to God. This is not just having the idea of God, but what ways of thinking can support the idea of God.

Dedekind offers an argument for the infinity of thought, which is quoted in Potter (2004, p71). My interpretation of his argument is as follows. Consider S as the set of objects of my thought, then $s \in S$ is an object of thought. s' is the thought that s can be an object of thought. $s' \in S$ as s' is also a thought. Also $s' \in S'$, where S' is the set of all thoughts that are thoughts about objects of thought. Now $S' \subset S$ (S' is a proper subset of S) because there are thoughts, s in S, such as my own ego and other thought processes that I do not think about. If a, b $\in S$ (a,b are objects of thought) there is a mapping ϕ (which makes s an object of thought) that takes $a,b \in S$ into $a',b' \in S'$. That is, ϕ is a 1–1 mapping between S and S′. That is, S is in a 1–1 correspondence with a subset of itself, which according to Dedekind's own definition, makes S infinite.

Here we are not talking about infinity so much as talking about the infinity of thought. Anselm's Definition requires that we be able to go into an infinite sequence. That is, no matter what thought of God we come up with, we can always come up with a greater thought. Now a greater thought may be a more inclusive thought (its extension involves more instances), or a greater thought may include more levels of thought (and language) than is contained in previous levels, as in the cumulative hierarchy. That means, for instance, that the beauty of God is compared with more and more things and is still more beautiful. Or that the beauty being considered is greater than any previous consideration of beauty in the sequence.

I need to consider why we do not need to go into the transfinite beyond the power of the continuum with Anselm's Definition. One argument is the use of the Lowenheim–Skolem Theorem.

8.2 The Lowenheim–Skolem Theorem

Lowenheim obtained the initial form of the theorem which stated 'that if a formula of the pure functional calculus of first-order containing no free variables, is satisfiable at all then it is satisfiable in a domain of individuals which is at most denumerable' (Runes (1960)p. 184). Alonzo Church (in Runes (1960)) continues that Skolem obtained the generalization that, 'if an enumerable set of such formulae are simultaneously satisfiable, they are simultaneously satisfiable in a domain of individuals at most enumerable'. This is a puzzling result for set theory, because under the usual interpretation, set theory asserts the existence of the non-denumerable infinite. Church continues in this article, by saying that any set of postulates designed to postulate about the non-denumerable will have an un-intended interpretation in the enumerable. Skolem, in an article reproduced in van Heijenhoort (2000) outlines his proof of this finding and his explanation of the apparent paradox. Briefly, Skolem places a first-order proposition in what he calls normal form where first the universal quantifiers and then the existential quantifiers are placed to the left of the proposition. The rest of the formula consists of class and relation symbols. Assuming that the proposition is consistent, by a process of infinite induction on N, it is explained how the proposition is satisfied by each of the integers so that ultimately, 'we can obtain as "limit" the fact that the first-order proposition is satisfied in the domain of the entire number sequence' (ibid.,p. 294). The relativity of set theory can also be expressed by saying that what is non-denumerable in one model is denumerable in another (Benacerraf (1985), p89).

How does this relate to Anselm's Definition? The question arises as to why we do not need to go into the transfinite pursuing the thought sequence? My initial answer is that the transfinite is not required in Anselm's Definition. Anselm, in fact, stays in the sequence. Anselm is being a constructivist. According to Anselm no matter what idea we offer in the thought sequence (or an idea in no thought sequence), God is always beyond that thought. So the sequence is going somewhere and, in some sense, has a limit thought which is only ever approached. Anselm's Definition says that 'God is …' . He is not saying that God is a thought sequence, nor even a thought, but that God is located by thought. The iterative nature of sets, I claim, models this thought sequence. We do not have to go beyond ω, because, according to the Lowenheim-Skolem theorem, there exists an enumerable interpretation of the non-enumerable set domain. It may be countered that the enumerable interpretation is in the domain of integers. Skolem's proof lay in satisfying the proposition in the realm of integers starting with 1 and proceeding inductively on N. If there is a 1-1 correspondence between the integers and thoughts governed by the > relation, it seems feasible, but tedious, to produced an enumerable domain of thoughts. And this is all that is required to satisfy Anselm's Definition.

8.3 Anselm's Definition with Multiplicity and Absolute Infinity.

I have used the ordinal sequence of numbers as a model of Anselm's thought sequence. However, it is known that the ordinal sequence is transfinite and remote infinite numbers such as inaccessible cardinals can be conceived. Anselm's Definition only requires the first ordinal ω. Attempts to restrict the endlessness of what Cantor devised have been considered. The following is Cantor's attempt to posit an Absolute Infinite, which would terminate the transfinite sequence.

Naïve set theory, initially developed by Cantor, produced the well-known paradoxes which were created by the 'naive' use of sets, in particular, the set of all sets. Von Neumann, among others, proposed a way to overcome this difficulty. This way was to distinguish between sets and classes. Only sets could be members of classes. Sets could be members of sets but classes were not to be the members of anything. This approach avoided the known paradoxes but there is no guarantee that all paradox is avoided.

The effect of classes is to stop the cumulative hierarchy, that is, stop the building of sets. However, Cantor had other ideas about truncating the cumulative formation of sets. Sets were built by the power set operator. In the transfinite this meant that the next transfinite cardinal after $\aleph_0$ was $2^{\aleph_0}$. At least this was Cantor's conjecture and it is known as the Continuum Hypothesis, where $\aleph_0$ is the set of all integers and $2^{\aleph_0}$ is the set of all the Reals (the continuum).

In a letter to Dedekind in 1899 (Van Heijenoort, p113) Cantor speaks about multiplicities. A definite multiplicity is a totality of things. Rather indistinctly Cantor defines two types of multiplicity. Either all the elements of a multiplicity can be conceived of as "together" (as one finished thing) or not. Those multiplicities that have a conceivable unity are called by Cantor consistent multiplicities; those that are not a conceivable unity are called inconsistent multiplicities.

According to Cantor, a consistent multiplicity has no conceivable contradiction and its elements can be gathered into ‘one thing’. This Cantor calls a set. Two sets whose elements can be placed in a one-to-one correspondence are said to be similar and to have the same cardinality. Consistent multiplicities can be described as simply ordered, similar or well-ordered.

A simply ordered or well-ordered multiplicity is a sequence. Every part of a sequence is a sequence. If a sequence F is a set then the type of F is its ordinal number and, having a least element, F is called the type, or size of this well-ordered set. The size of the set is the general notion applied to a set and all sets similar to it.

Cantor then considers Ω the set of all ordinals. Ω is a simply ordered sequence of ordinals

$$\Omega = 1,\ 2,\ 3,\ \dots\ \omega,\ \omega+1,\ \dots\gamma\dots$$

Cantor forms another ordinal sequence Ω' by prefixing the list with 0. Ω' is a well-ordered set of ever-increasing ordinals where any ordinal is the set of all those ordinals before it in the sequence

$$\Omega' = 0,\ 1,\ 2,\ 3,\ \dots\ \omega,\ \omega+1,\dots\gamma\dots$$

Cantor then argues that $\Omega'$is not consistent. To any well-ordered set there corresponds an ordinal number. If Ω' were consistent there would correspond to Ω' an ordinal ζ which would be greater than Ω. But Ω is defined as containing all ordinals so Ω both does and does not contain ζ. (This argument became known as the Burali-Forte Paradox of the greatest Cardinal). Cantor calls this inconsistent multiplicity Ω an absolute infinite multiplicity. He says ‘the system Ω of all numbers is an inconsistent, absolutely infinite multiplicity’ (ibid, p115).

Cantor goes on to develop a theory of transfinite numbers where an aleph ($\aleph$) is the cardinal number of the first infinite ordinal

$$\aleph_0 = \overline{\omega}_0.$$

The number class of all ordinals corresponding to the cardinal c is denoted by Z(c). The number class is the set of all ordinals corresponding to the same cardinal – that is, having the same size. Therefore

$$Z(\aleph_0) = \Omega_0$$

which says that the number class of all ordinal numbers corresponding to aleph null $\aleph_0$ is Ω_0. Cantor attempts to show that the system of all alephs (transfinite cardinals, which he calls tav) is similar to Ω and therefore inconsistent or absolutely infinite.

Van Heijenoort points out a weakness in Cantor’s proof and indicates that the axiom of choice, not recognised by Cantor, is the basis for a better proof.

I wish to establish an analogy or parallel between Cantor’s theory of multiplicities and Anselm’s definition.

The notion of an inconsistent multiplicity suggests a discontinuity in the sequence of cardinals. That is, we go as far as we can conceivably go (the ordinal sequence) and then we posit a limit. In that the limit is not a term of the infinite sequence, the set of ordinals (or cardinals) cannot be viewed as a consistent unity. To include the limit creates the inconsistent multiplicity. Similarly with Anselm the sequence of thoughts can be viewed as a unity or a consistent multiplicity. However, the limit is not a term of the sequence: God is not a thought. So to posit God as the limit to the thought sequence creates the inconsistent multiplicity or absolute infinity.

In this kind of discussion it appears that Cantor is seeking to come to grips with the endlessness of the transfinite sequence of ordinals. It is as though he has created something that he does not know how to finish. If every infinity can be treated as a completed infinity one can readily develop an infinite sequence of completed infinities *ad infinitum*.

It would seem that Cantor's attempt to conceive of an absolute infinity is an attempt to end this sequence of infinities. The system tav of all alephs is an attempt to treat the transfinite cardinals as a completed set. The absolute infinite is created by the inconsistent multiplicity. The inconsistent multiplicity is created by the "not togetherness" as the limit of the alephs is not a member of the sequence.

Anselm's definition acts in a similar way. The concept of God is used to complete the infinite set of the thought sequence. The concept of God creates the inconsistent multiplicity – the item that is not part of the unity.

One is tempted to claim that the Absolute Infinite, which is the set tav of infinite cardinals, corresponds to God.

But God is not a set or even an infinite collection of sets. Cantor can use the Absolute Infinite to stop the cumulative iteration of the endless generation of sets because he claims that 'the system tav of all alephs is nothing but the system of all transfinite cardinal numbers.' (ibid, p117), which is a completed infinity. By this means Cantor obtains completion.

Those who reject the axiom of choice would presumably say the transfinite sequence (if it exists) goes on forever on the pattern of the formation of the first transfinite number. The argument of multiplicities seems to be Cantor's way of showing how the transfinite number system he himself devised can be contained. It would seem that by being adventurous and going beyond the first infinite ordinal and creating an infinite sequence of infinite ordinals, Cantor has unleashed a process he cannot contain. In some sense this infinite sequence of infinite ordinals needs to be conceptually completed. This is the whole point of limits; limits complete any infinite process making the process conceptually usable. Therefore Cantor needs completed infinite sets and so does Anselm to complete any thought sequence.

Similarly, Anselm is offering God as the way to curtail the infinite thought sequence.

Here I have used these thoughts of Cantor to attempt to explain how an infinite sequence may be completed. In a similar fashion Anselm's definition may be viewed as an attempt to curtail an infinite thought sequence.

It all hangs on the possibility of completed infinite sets.

8.4 Anselm's Definition and the Axioms of Infinity and Choice.

As we are looking at methods of generating Anselm's thought sequence we have looked at number sequences with limits and more specifically ordinal sequences with limit point ω. We have also looked at the cumulative and constructible hierarchies as ways of generating a thought sequence. The cumulative hierarchy, using the power set operator, gave us the maximum number of sets at any level which, when pushed to the infinite, becomes huge. The constructible hierarchy reduces or minimizes the complexity or richness of each set in the sequence.

We are positing Anselm's Definition as a thought sequence, which is described by the iterative concept of set. This concept states that sets can be built up from other sets in a systematic way, say by using the power set operator. The members of the set appear before the set that contains them. This helps to avoid impredicative sets. Sets can be associated with each level of the constructible hierarchy of sets. Let us associate with each level of sets a language. The language at level V_i deals with the objects, sets of objects and relations between them built by the iterative process. Any language may refer to any object in a previous language. The languages differ in the domains of discourse over which they are permitted to range. This cab be expressed as a Type Theory where each level in the conceptual hierarchy can only speak about current or lower types or concepts. This is done to avoid impredicative sets or self-reference by sets. This gives us a recursive hierarchy of languages where each level provides a language with restricted quantification for its variables. Anselm's Definition does not require a language sequence, only a thought sequence. For the sake of concentrating on one attribute, there can be a minimal selection of one thought per level (or language domain). This selection is what the Axiom of Choice guarantees.

There are two Axioms here that we require. Let me note that Axioms of Set Theory are used to guarantee the formation and existence of certain sets.

Briefly the Axioms of ZF Set Theory may be summarised as follows with emphasis on the assertion of their existence.

1. There is a set. Sets with the same members are the same (Extensionality).
2. There is a null set (no members) (Elementary sets).
3. There is a subset of a set (Selection of subsets).
4. There is a power set (all subsets of a set).
5. There is the union of a set (all members of members of a set) (Union).
6. There is a choice set (Choice).
7. There is an infinite set (Infinity).

Fraenkel (Fraenkel 1984) introduces other axioms including the Axioms of Pairing, Replacement and Foundation. There are various collections of Axioms for Set Theory in the literature. The axioms can be arranged according to what one wants to do in the Set Theory being considered.

I would suggest an arrangement of five axioms to meet requirements for Anselm's Definition as follows.

1. There is a set

2. There are elements of sets

3. There are operations on sets

4. There is an infinite set.

5. The Axiom of Choice.

Axioms fulfil two functions. One function is to get a Set Theory off the ground. The other function is to restrict what is possible and avoid the known paradoxes. Restricting what is possible need not actually be an axiom but the restriction can be on specific operations that are possible on sets.

I would explain these five axioms as follows.

1. There is a set.
   This allows for set formation. There is the possibility of a set as an object of inquiry. Sets are conceivable objects of thought.

2. There are elements of sets.
A set can be viewed as a collection of elements. There can be no element – the null set. There can be equal elements – equal sets have the same members. There can be sets as elements of sets – subsets and supersets.

3. There are operations on sets
Things can be done to sets. Sets are objects to be manipulated. Such operations include membership, power set,

infinite sequence, inclusion, union, selection or choice, subset, intersection, superset and ordering.

4. There is an infinite set.
In 3 above we listed the infinite sequence operation. The Axiom of Infinity can be expressed as

There is a set Z with $\varnothing \in Z$ and for any $x \in Z, (x \cup \{x\}) \in Z$, giving $\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \ldots$ and $\{\varnothing\} \cup \{\{\varnothing\}\} = \{\varnothing, \{\varnothing\}\}, \ldots$

The operation $x \cup \{x\}$ allows the element x to be added to itself expressed as a set. As we run over values of x we can generate an infinite sequence. Z is this set at any point of its generation. This means Z can be finite or potentially infinite. $x \cup \{x\}$ is a recursive operation that generates a potential infinity. Anselm’s Definition can use a completed infinite set. We have to make the conceptual move from potential to actual infinity to actually obtain our concept of God. This gives us a limit and the limit so required denotes God. $x \cup \{x\}$ indicates the process, Z indicates the set. It is the completed infinite set that is our ultimate concept of God.

5. The Axiom of Choice allows a thought sequence to be selected.

I need to point out here that I am not claiming that any infinite sequence points to God. Anselm's Definition is a method or algorithm that, if we are thinking about God, can take us in the direction of God. Set Theory and Mathematics, as such, are not about God, even as we may say that the creation, as such, is not about God. What I seek to argue is that, in taking up the idea of God, (in starting the thought sequence with a God attribute), there are established ways of thinking that are consistent with a belief in God. Such ideas are consistent with the existence of an object that I call God.

The Axiom of Choice is also part of the construction of Anselm’s Definition. If we consider the infinite sequence generated by the Axiom of Infinity we can generate a recursive set of languages modeling the ordinal sequence of numbers. However, Anselm’s Definition does not require a whole language only a thought in a language. Therefore we need a mechanism to select a thought from each potential language that is generated. The Axiom of Choice provides this selection. If we say that the sequence of languages has restricted quantification so that the variables in a language statement can only refer to concepts defined at a previous level, then we can avoid contradiction at least of the impredicative type. So the thought sequence is consistent. Gödel proved in 1939 that the Axiom of Choice is consistent with the other Axioms of ZF, so the addition of the Axiom itself to ZF does not create contradiction.

The Axiom of Choice has many equivalent forms (Rubin and Rubin (1963)). One form is as follows.

Given a set S with subsets $s \in S$ where all the s are non-empty and all are disjoint, then a selection can be made of one element x from each set s. The set C ($x \in C$) is the Choice Set and the selection function F($F: \cup S \rightarrow C$) maps members of the members of S to C.

This definition of the Axiom of Choice can be related to Anselm’s Definition as follows.

The set C selected from the set S is the thought sequence selected from the cumulative hierarchy of thoughts or languages.

This selection attracts the standard criticism of the Axiom of Choice from the constructivists. That is, that, if the sequence is infinite (as Anselm's Definition requires) the selection cannot actually be performed. At this point we need to emphasise the purely existential character (Fraenkel 1984, p68) of the Axiom of Choice. As Fraenkel notes, the Axiom is not claiming that an infinite construction is possible. What the Axiom of Choice provides is the guarantee (if the Axiom is accepted) that the choice set exists.

The need for the Axiom of Choice in Anselm's Definition may be shown as follows, following a well-known example by Bertrand Russell (Russell (1970) p126). Russell supposes that a millionaire buys $\aleph_0$ pairs of boots and he always buys a pair of socks with a pair of boots. The question is: how many boots and socks does he have? Because $2 \times \aleph_0 = \aleph_0$ we cannot simply double the infinite cardinal. Another way is to put each pair of boots in a 1-1 correspondence with the natural numbers up to $\aleph_0$. This will work for the boots for we may select either the right or the left hand boot and not fear doubling up on boots (or using the same pair twice). However, this cannot be done for socks because we cannot make a selection of only a left or right sock because they cannot be so distinguished. Therefore we cannot make a selection unless we assume the Axiom of Choice which says a selection can be made.

We relate this to Anselm's Definition as follows. We have a thought sequence ordered by the > relation. In the hierarchy of thoughts there will be a set of thoughts at a particular level because the power set operation generates more thoughts at each application. Anselm's Definition allows for discrimination between thoughts only by the > relation. Therefore we can only distinguish between thoughts across levels not within levels of the cumulative hierarchy. How do we distinguish thoughts within a level? Thoughts will be like socks (there is no left and right foot sock) so, in that there is no way to distinguish between them, the > relation is all the Definition provides. Therefore we need the assumption that, in principle, a selection can be made to construct a method to establish a thought sequence to an infinite limit.

The axiom of choice asserts that for any (infinite) set of sets there always exists a set that consists of one element from each of the other sets.

If we regard the thought sequence as a sequence of sets, t_i, so that $t_i < t_{i+1}$ (where $<$ is a relation) or $t_i \subset t_{i+1}$ (where $\subset$ is an operator) and where each t_i is selected from the set of available thoughts at level i, then we can construct such a thought sequence on the basis of the axioms of Set Theory.

In fact, the axiom of choice guarantees that a Thought Sequence is always obtainable. The existence of the choice set says that given the constructive hierarchy a selection of a thought at any level can always be made. The axiom does not guarantee which thought can be selected but it does guarantee that a thought can be selected and that a thought sequence always exists.

It is also interesting to note the long debate over the appropriateness of the Axiom of Choice. However, this debate has subsided, not because there is some new compelling proof of its existence, but because it has proved to be so useful and, if it is denied, much mathematics is unavailable. Analogously, the idea of God remains because it is useful and supplies conceptual closure on the boundaries of thought.

In the above I have intended to show how well-established methods of thinking in Set Theory can be used to guarantee the construction of a thought sequence. No particular thought is specified but the structure is possible. This use of Set Theory is stronger than analogy or illustration of technique. I claim that this use of Set Theory is (conceptual) evidence of workable thought structures that can accommodate and work with the plausibility of God’s existence.