13 God and Complexity

13.1 Collapsing the range of Limit points

The thrust of Anselm's Definition is to generate a thought sequence that puts God beyond what we can think. Much talk about God can be an attempt to make God more inaccessible and remote, bordering on the inconceivable. I wish to counter this trend with the argument that the descriptions of God are collapsible. All the incomplete sequences can be subsumed under the word ‘God’. An infinite sequence can be collapsed into an interval. The infinite ordinal sequence can be summed up in the term ω. This is the concept of the completed infinity. Conceptually, a completed infinity can become an object that in turn can be part of another infinite sequence. So we can have transfinite arithmetic such as $\omega + 1$ and $\omega + \omega$. This also means that the idea of an infinite sequence can be expressed as the set of reals between 0 and 1 or any other segment of the real line. An infinite geometric sequence between 0 and 1, with first term ½ and geometric ratio ½ has a limit of 1 (using a/(1-r) with a= 1/2 and r= 1/2). It does not matter which infinite sequence we refer to in order to illustrate Anselm's Definition, as long as the sequence can be said to have a limit. There can be two kinds of limit. 100 can be said to be the limit (or least upper bound) of the first 100 integers. If we regard the numbers 1 to 99 as a thought sequence then we have a model of Anselm's Definition. However, we can easily think of more thoughts (more numbers) and so we have not approached what we would regard as a greatest thought, or a thought that was the furthest that we could go or the greatest that we could think. Therefore a better illustration of an infinite thought sequence is an infinite sequence of what Sierpinski calls 'of the second kind' (Sierpinski (1965)(p 274)). A limit number of the second kind is a number in a sequence without an immediate predecessor number. If we liken the numbers to numbered thoughts then we have a limit thought that has no predecessor. Once we have the limit we are out of the sequence and we are viewing the sequence as completed. If there is no predecessor thought, that means that the limit thought is not an immediate successor of any thought in the sequence. If we are in the thought sequence we are forever in the search of the greatest thought denoting God. Any of the thoughts in this thought sequence may be used to denote God, with the proviso that all that this thought sequence can say about God may not be all that can be said about God. For instance, we can go on forever exploring the depth and relevance of the goodness of God and never fully understand the power and extent of God's behaviour as an expression of goodness. We may use thought 'good_{785}' to express God's goodness on some scale of goodness, but if we collapse the sequence and speak of limit thought 'good_{ω}' then we are more accurately inferring what we want to say about God.

I want to explore the idea of collapsing the ideas of God produced by thought sequences. The problem arises that if each thought sequence produces a limit thought, do we have a 'god' at each limit? If we are arguing for 'god', do we have a god for beauty, another god for goodness, another god for wisdom etc.?

If we obtain a limit point from the thought sequence, the question arises as to whether the limit point is unique? If one claims that there is a limit point to every thought sequence, does this imply that there are as many limit-thoughts as thought sequences? Is there a limit-conceivable for every conceivable thought sequence? I need a concept with respect to Anselm's Definition that solves the problem of multiple limit points. If every thought sequence illustrated by an attribute creates a different thought limit point then how can we obtain only one greater-than limit point? Otherwise there would be a god for every attribute. One way out of this problem is to claim that the attribute does not generate God, it only points to God. Another way out is to say that any limit points converge into one point. I am suggesting that the feasibility of God denoted by the limit point can lie in this concept of collapsing many points or sequences into one point.

Anselm's goal is to have only one limit thought or point for all thought sequences that are about attributes of God. I argue for a convergence of sequences where all the limits are in fact one limit. The convergence represents the idea that the descriptors (or attributes) of God cohere in the idea of God. This is saying that 'God' is an idea that subsumes other ideas relevant to it. This may be likened to the great circles on the surface of a sphere converging to a point of intersection, such as the lines of longitude intersecting at the North Pole. This is an attempt to answer the question of why should limit thoughts be referring to God? Cannot limits just be limits? Of course limits are just limits, but we are using limits to structure thought at the edge of the conceivable. The theory of limits is a way of talking about God and a conceivable way of approaching God. I do not have to end up with God. I can end up with the greatest conceivable goodness, beauty, holiness and wisdom … I can definitely end up with a conceivable collection of conceivable limits of conceivable virtues. That I collapse them into the idea of God is an intellectual technique. Instead of talking only about thought sequences and individual descriptors, I can claim that I have indicated a possible existent, namely God. Thought can do no more than be a descriptor or indicator. Now that thought has given me a concept (of God) can the thought give me existence? Can I squeeze existence out of thought? I do not think so; that is ontological thinking. But can thought corner existence (an existent)? Can thought deliver me with a rational, intelligible, consistent and apt concept (of God) that is not only rational in itself, but can be rationally argued for and is consistent with knowledge already attained? I believe that it can. This is done by bringing other arguments into consideration, which may be ethical, historical, personal or cultural. Even if God is the very best of our ideas, it does not follow that the idea of God implies its own instantiation. This would be theory-initiated existence. In my account, the existence of God is prior to the thought of God, because knowledge about God is based upon the existing creation. However, thought and intelligibility may lead us to what already exists. I can also use thought to more deeply embellish the ideas of God and their ramifications.

I am suggesting that, ultimately, the predicate '- that than which nothing greater can be thought' can have only one referent although approached by various thought sequences.

In speaking of the limit point I take it only as far as ω in the ordinal sequence. If we describe the limit as the least upper bound this is suggesting that there are other upper bounds of which the limit is only the least. Anselm's Definition does not allow for or need this conceptual provision. I claim that one limit is sufficient. With further limits we are simply replicating the limit procedure.

Following this line of argument we can say that the peculiar nature of the limit (God – greater than any conceivable thing) is that it appears that the descriptors are best described to be converging. There is a closure among descriptors in the idea of God. We may argue that the ω limit is only one limit not many limits. There are not an infinite number of ω points. There are many thought sequences approaching ω-limit thoughts, (in the case of God) as many as there are descriptors of God. But as every thought sequence approaches its ω-limit thought, is there a convergence to the one ω limit across sequences? Is this an example of the tricks played by infinity on infinitely dense finite intervals? If an infinite set is in a 1-1 correspondence with a subset of itself, can all the subsets progressively degenerate (collapse) into the ω point (the idea of God)? We could then claim that, in the limit, the descriptors or thought sequences do converge to ω. But in such a process we have lost our sequence. If this is claimed for infinite thought sequences it must apply to all infinite thought sequences. Hence we have the Gaunilo-type critique of infinitely conceivable objects other than God. What is the answer here? I admit that other infinitely conceivable objects can be conceived including maximal evil. However, whenever I am dealing with God I will be using God descriptors, which I claim are radically different from evil or the purely material such as islands. This is a question of what I am talking about not how I am talking about it. We may have limit concepts of anything that can be conceived in a limit fashion, but I claim that the idea of God (with its descriptors) is beyond any other limit concept (and its descriptors). So the argument is not only along limit thought descriptors (such as the descriptors of any object conceived in the limit as in the greatest conceivable island), but across limit thought descriptors for different objects. That is, we are comparing limit thought descriptors as to what they are describing and then maintaining that God is the greatest among the limit conceivable objects. So I build the concept of the infinitely maximal being, God, which is the greatest amongst the limit conceivable beings or objects. Once I have attained this idea, I do not think that I can claim that it exists because I can think it, but I can examine the idea in the context of other ideas and decide on its plausibility, consistency and relevance to a conceivable universe.

This suggests the following conjecture.

13.2 The Anselm One Point Conjecture

> In the limit, all the descriptors subject to the greater than operator GT (thoughts compared by the > relation) collapse into one point at infinity, ω. This one point indicates God.

It must be added that I obtain the concept of God only if I use God descriptors. If I use other descriptors, such as evil, I obtain another maximal concept, a maximal evil being. If I really want the maximal being, I then have to decide among maximal concepts of being. This can produce another thought sequence whereby I choose a maximal being from amongst collections of concepts of maximal beings.

I offer the following points as items for assembling an argument for the conjecture.

1. The uniqueness of the object (God) counteracts the traditional ontological arguments that claim perfection implies existence. If one claims that a limit thought sequence produces a perfectible object then every thought sequence produces a perfectible object. If perfection implies existence then the perfectible exists. Hence Gaunilo argues that on these assumptions he can produce a perfect island or, in principle, a perfectible anything else. So the argument for uniqueness counters the multiplicity of perfectibles. An argument for uniqueness is the attempt to make God the maximal of the maximal beings or objects. But we also need the limit thought attributes of God to converge to God.

2. Arguing by mathematical analogy, we can consider the projective plane π in which parallel lines are defined. (Horadam (1970), p207). If we choose any projective line we can call it the line at infinity l_∞. In the projective plane all parallel lines in the plane π meet on the line at infinity at a point at infinity P_∞. Using this as an analogy for Anselm's Definition, we can say that all limit thought sequences when extended into infinity, meet on the line at infinity, l_∞, at a point at infinity, which is the limit thought of God.

3. Another mathematical analogy from topology is as follows (Simmons, p163). The point at infinity is the point added in the 'one point compactification' of the complex plane. This extended plane may be identified with a sphere of which it is the conformal image under stereographic projection. The point at infinity corresponds to the pole of the projection.

   That is, to the space S we add an object ∞, not in S, to form the set $S_\infty = S\cup\{\infty\}$. The addition of ∞ to S is called the 'one point compactification' of S by ∞. Analogously for Anselm, the thought sequences as lines on S converge to the point at infinity, which is a limit or ideal point which can denote the limit thought of God. Analogously, ∞ is not a member of S even as the limit thought of God is not an element of the thought sequence, but is used to complete the sequence.

4. Following on the idea of possible or conceivable worlds, which are described by true propositions, there is that proposition which is the conjunction of all true

propositions about a maximal world. God can be described by that proposition which is the conjunction of all true propositions (in limit thought form) that are true statements about God. This one proposition is a convergent, or one point description of God, even though it has many parts.

5. It is to be remembered that Anselm's Definition is a formal piece of reasoning. That is, the argument does not rely on discussion of attributes, which are interpretations or illustrations of the formal argument. If we regard Anselm's Definition as a model or interpretation of the ordinal sequence reaching a limit in ω, then we are dealing with only one limit. There are not many ω's but only one. Therefore the limit is only one point, as a formal property of the ordinal sequence.

Another way to view a reduction or convergence in the multiplying of infinities is the Lowenheim-Skolem Theorem. The challenge of the transfinite created by completed infinities and the proliferation of transfinite ordinals and inaccessible cardinals, is that if I apply Anselm's Definition to infinite sequences where does the application finish? Does the commitment to infinite thought sequences force me to continue on into the transfinite and, if so, what can this mean? I argue that Anselm's Definition does not require me to use any infinite process beyond the ordinal ω and cardinal $\aleph_0$. If I stay inside the thought sequence I remain strictly in the finite and the constructible. If I go outside the thought sequence and see it as completed with a limit, then I have simply placed myself at the completion of the first infinite ordinal ω. The argument and design of Anselm's Definition does not need to go any further. Because I am looking for a pattern for thought and not for a particular mathematical result dependent on particular numbers, I can simply stay in the initial infinite sequence. I claim that the Lowenheim-Skolem Theorem adds weight to this decision.

13.3 The Lowenheim-Skolem Theorem again

The Lowenheim-Skolem Theorem can be expressed in various ways. Skolem expanded an earlier form of the theorem that had been proved by Lowenheim. Lowenheim had shown that 'If a first-order proposition is satisfied in any domain at all, it is already satisfied in a denumerably infinite domain' (Van Heijenoort (2000), p293). Skolem expanded this finding to produce the following version of the theorem: In a first-order theory, 'if the axioms are consistent, there exists a domain B in which the axioms hold and whose elements can all be enumerated by means of the positive finite integers' (ibid., p293). Another way to express this is that any class of well-formed formulae of the propositional calculus, which has a model (or interpretation), will have a model (or interpretation) with a denumerable domain. I take this to mean that whatever first-order propositions can be satisfied in the transfinite can also be satisfied in the denumerable. Another version of the theorem is that if a first order theory has a model, then it has a denumerable model (Magaris (1990), p166).

Skolem comments. 'By virtue of the axioms we can prove the existence of higher cardinalities, of higher number classes and so forth. How can it be, then, that the entire domain B can already be enumerated by means of the finite positive integers?' (ibid., p295). This gives rise to Skolem's paradox which highlights the 'paradox' that the non-denumerable set can be modeled by a denumerable set, provided the axioms that are modeled are consistent. Cantor's theorem specifies that the cardinality of a power set of a set is greater than the cardinality of the originating set. This means that larger and larger sets can always be guaranteed. However, the transfinite can be modeled by the denumerable infinite. Hodges (Hodges (1997), p69) points out that Skolem disliked uncountable structures and wanted to show that they were not necessary. 'He proved that for every infinite structure B of countable signature there is a countable substructure of B which is elementary equivalent to B' (ibid., p69). This means that there are countable models of the sentence 'there are uncountably many reals'. If, therefore, we can have countable models of uncountable models, I claim that this means that if Anselm's Definition is pushed into the transfinite there will be an equivalent countable model for it. Therefore Anselm's Definition can suitably remain in the countable infinite using ω as the limit-thought indicating God.

Potter makes a comment about Skolem's paradox as follows. In speaking about the paradox he says that using the submodel version of the Lowenheim-Skolem Theorem, we can deduce that there is a countable set M such that models set theory. Yet, as a model of ZF set theory, all theorems of

ZF are true in M. M has a member such as the power set of the natural numbers which is not countable. This apparent paradox can be resolved, according to Potter, by a change in the range of the quantifiers, which is a variation 'sufficient to change the extension of the predicate "countable"' (ibid., p241). Potter then says 'Thus set theory has (assuming always that it is consistent) a model M which is uncountable from within and countable from without' (ibid., p241). This is an exact description of Anselm's Definition. From within the sequence we never finish, the thoughts are uncountable or never fully enumerated. From outside the sequence, the limit is obtained, the sequence is completed and, in turn, may become an item in another sequence. However, this possibility is not required in realizing Anselm's Definition. As Potter points out, this does not mean that every possible set is really countable, but that a possible countable interpretation has been demonstrated.

13.4 Anselm's Definition and related Ideas

Anselm's Definition, as conceived as a thought sequence, can collect around it a number of related ideas. These ideas are listed and explained as follows.
1) The Axiom of Constructibility, that every set is constructible (Potter (2004), p253) indicates formally, that a thought sequence can be constructed. If we consider a cumulative hierarchy of sets, constructed by the power set operator, we generate a huge number of sets. The constructible hierarchy restricts the formation of sets so that, for a level L_i in the hierarchy, sets in the following level L_{i+1} will consist only of sets defined by

formulae whose quantifiers are restricted to range only over L_i. Godel showed that this notion of constructibility can be formalized within set theory. In fact, Godel showed that all the axioms of ZF set theory hold when all the quantifiers in them are restricted to the constructible hierarchy L. The axiom of constructibility also entails the axiom of choice (Potter (2004), p253).

2) The Lowenheim-Skolem Theorem gives us a countable model. As I have argued, this theorem allows us to work only in the countable thought sequence although there may be reference to the uncountable.

3) Well-ordering, that gives us a least element in the thought sequence. Anselm's Definition, as a formal definition, does not require us to use an actual attribute to understand the thought sequence. An attribute is used as an illustration, or application of the thought sequence. A well-ordering means that we have a least element in the sequence. If we are using an attribute it is important to be able to start with that attribute in some minimal form. If, for instance, we are using ‘goodness’ as an attribute to illustrate the thought sequence, then we need to start with goodness, even in a minimal form, rather than starting with evil as a lesser form of good and attempting to go from lesser evil to greater good. I do not argue that any thought can lead to God because God, as object, is discernible from other objects by means of attributes or descriptors. This argument means that we do need attributes to get to the right object, God. Does this discount the formality of Anselm's Definition? The thought sequence gives us the maximal being or maximized object. In Gaunilo-type objections, the purely formal can mean a maximized

island, or maximized evil being or idealized financial situation. Given a maximized anything, we then use the attributes to distinguish between them, assuming that the object can be maximized. God then, becomes that maximized being exhibiting maximized descriptors that are characteristic of God. The key question becomes, why is existence granted to one idealized object and not the other? Ontological thinking obtains existence from the nature of the attribute, so that the attribute of perfection implies existence. Anselm obtains existence from ‘existence in reality’ being greater than ‘existence in the mind’. I argue that existence is an event that is not derivable from thought but may be discoverable by means of thought. The Well-Ordering Theorem, that every set is well-orderable, is equivalent to the Axiom of Choice (Levy (2002), p161).

4) Zorn’s Lemma, obtains a maximal element in an ordered set in which every chain has an upper bound. A collection A is called a chain if it is totally ordered by inclusion. Zorn’s Lemma says that if every chain in a partially ordered set P has an upper bound in P, then P has a maximal element. I claim that an attribute can (often) be conceived as a sequence of greater and greater aspects of that attribute. So that we can conceive of greater and greater goodness or wonder. The goodness and wonder may be of greater intensity, significance or relevance to a greater area. I claim that the levels or gradations of the attribute are inclusive in that any lower level of attribute is implied in any higher level of attribute. Zorn’s Lemma is equivalent to the Axiom of Choice (ibid., p162).

5) The Axiom of Choice, guarantees that a thought selection can be made from a constructed thought sequence. The Axiom of Choice has created much controversy mainly because of its application to the infinite. The axiom states that for any set A of disjoint, non-empty sets $a \in A$, there exists a set c which consists of exactly one member from each a. Rubin and Rubin (1963) and Moore (1982) detail the history and the equivalents of this axiom. The axiom is equivalent to the assertion that every set has a choice function. The choice function f is the function that chooses the $a \in A$ so that $f(a) \in c$. In application to Anselm's Definition, we may say that as a thought sequence is generated, say as a constructible hierarchy, there is always a choice function that makes possible a selection of a thought from any particular level of the thought sequence. Axioms, such as the Axiom of Infinity, are devised to make the construction of sets possible. Gödel has demonstrated that the Axiom of Choice is consistent with the axioms of set theory, but Cohen has shown that the axiom is independent of the axioms of set theory, which means that the axiom is unprovable in ZF.

13.5 Argument from Complexity

Is there an argument for the existence of God that is stronger than saying that the idea of God is only consistent with other ideas? It has been my attempt to show that the idea of God is consistent with various kinds of mathematical arguments and structures. I shall attempt to argue for existence using an assumption of what I shall call completeness. I am aware that completeness is a technical term in Logic, however, I wish to argue from the completeness of the creation. That is, that the creation is not only created ex nihilo and spoken but is also complete. God has finished all God’s works. This does not mean that everything remains the same, but that there is a conceptual limit to the variability within the creation. What can happen to things depends on the existing, created relations between them. The argument from complexity states that a system cannot create a level of complexity that it does not already have. To contain the idea of God, the conceptual system must already have the capacity for the level of complexity needed to express the required idea of God. If the creation is conceptually complete, is this akin to the idea of a conceptual boundary? Does Anselm's Definition push us up against a conceptual boundary beyond which we cannot conceive?

Barrow (Barrow (1992), p137) speaks about the complexity of a sequence and the computer program that prints it out. Associated with any sequence (say of numbers) is a program to generate it (print it out). The less complex (more ordered) the sequence is, the shorter the program needed to print it out. The more random the sequence the longer the program needed to print it out. Until the most random sequence is the same length as the length of the computer program to print it. A measure of the complexity of the sequence is the length of the program (in computer bits). A program cannot print out a totally random sequence longer than (more complex) than itself. Randomness in the sequence expresses the complexity of the sequence. A totally random (complex) sequence is maximally random for that sequence length. A measure of the complexity of the sequence is the length of the program. The program cannot print out a totally random sequence longer (more complex) than itself. That is, the complexity (length) needs to be in the machine (system). The system itself, as the program, cannot create a complex sequence more complex than itself.

Now we know that conceptually, things can be complex. Complexity about a system can be expressed by sequences of propositions about the system. Consider the most complex thing (sequence). This cannot be created by a less complex system. The most complex thing (sequence) has to be created by something at least as complex. Nothing in the creation can create the most complex thing. So something, at least more complex must exist for the most complex thing to be created (to exist). This most complex (Creator) source is God.

An objection can be made that we are only really dealing with sequences and not objects. Sequences expresses the complexity contained in the objects. The fact that complex things exist and that sequences can portray the complexity means that complexity in objects is conceivable and a way to manipulate objects.

13.6 Is there a most complex thing?

I would claim that if the creation is conceptually complete then there must to be a most complex object. If complexity cannot be created by less complex agents, then there has to be an external source of complexity and a most complex thing. If there is not one most complex thing, then let there be many equally complex things (sequences). Let these be maximal sequences (beings). If all these equally complex sequences exist, they will be of equal length. Let there be a conjunction (concatenation) of all equally complex sequences (objects). This is the maximal conceivable complex sequence. It cannot be created by the system. If the system (concatenating agent) is as complex then the procedure is repeated. That is, the equally complex system can create many equally complex sequences, which in turn can be concatenated by some more complex agent. Either we have an infinite regress or we allow for a more complex agent outside the system. The agent creating the most complex sequence is the most complex concatenating agent, which is a description of God. Presumably this maximally complex sequence is a description of the conceptually concatenated complete creation. The creation is conceptually completed because it cannot make itself more complex than the concatenating agent (the Creator). If God becomes more complex then there can be, conceivably, a more complex creation. However, if God does not become more complex then the creation is maximally stable in complexity or on a complexity boundary.